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PROPAGATION OF PRESSURE WAVES IN TWO-PHASE FLOW

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Abstract—We deal with a pressure wave of finite amplitude propagating in a gas and liquid medium or in the fluid in an elastic tube. We study the effects of pipe elasticity on the propagation velocity of the pressure wave. Pressure waves of finite amplitude progressing in the two-phase flow are treated considering the void fraction change due to pressure rise. The propagation velocity of the two-phase shock wave is also investigated, and the behavior of the reflection of the pressure wave at the rigid wall is analyzed and compared to that in a pure gas or liquid. The results are compared to experimental data of a pressure wave propagating in the two-phase flow in a vertical shock tube.

1. INTRODUCTION

The flow of a mixed fluid consisting of two phases such as a vapor and liquid is more complex than that of a single phase fluid owing to an increase in the number of degrees of freedom for the flow. Simplifying assumptions are necessarily invoked to permit problem solution. A frequently used model incorporates mean values for the two phases to treat the two-phase flow as a single-phase flow. However, some two-phase flows cannot be solved using this model owing to large differences in the physical properties, such as density and compressibility coefficient between the phases.

The propagation of pressure waves is one of the characteristic phenomena of two-phase flow. It is known that the propagation velocity of a pressure wave of small amplitude in two-phase flow decreases very rapidly with increasing void fraction. Therefore, after passage of a pressure wave the pressure increases and a shock wave is generated owing to the increase of the sonic velocity. In the low void region, the change of the sonic velocity with void fraction is very large, and a stronger shock wave is generated than in the case of a gas only. On the other hand, the pressure wave in the liquid retains its initial form, both in the expansion and compression waves. As the liquid is almost incompressible, the elastic deformation of the pipe wall is large and affects the propagation velocity of the pressure wave.

This study deals with a pressure wave of finite amplitude propagating in a gas–liquid two-phase medium or in the fluid in an elastic tube. Additionally, pressure waves of finite amplitude progressing in the two-phase flow are treated considering the void fraction change due to the pressure rise. The propagation velocity of a two-phase shock wave is also investigated, and the reflection of the pressure wave at a rigid wall is analyzed and compared with those of pure gas or liquid to determine the effect of the virtual mass. Also, experiments on pressure waves propagating in the two-phase flow in a vertical shock tube were performed.

Investigations of the propagation of pressure waves in two-phase flow are divided into studies of the dispersion characteristics of sound waves (Stadtke 1968; Mori 1971) and studies of finite amplitude pressure waves (Campbell 1958; Henry 1968). In most of the latter studies, adequate conclusions have not been obtained because they compare experimental results to the sonic velocity without considering the finite amplitude of the pressure waves. This study considers analytically that the pressure wave has a finite amplitude, and compares the results with experimental data to determine the relation between the propagating velocity of a pressure wave and the sonic velocity in a two-phase flow.

2. THEORETICAL ANALYSIS

2.1 *General equations of the pressure waves in the elastic pipe*

Analyzing the propagation velocity of pressure waves in two-phase flow, we first study the effects of an elastic deformation of the pipe. The fundamental equations for the pressure wave which propagates in a uniform medium of density ρ, are, neglecting viscosity:

mass conservation law:

$$\mathrm{d}\{\rho u A\} = 0, \qquad [1]$$

momentum conservation law:

$$\mathrm{d}\{\rho u^2 A\} + A\,\mathrm{d}P = 0, \qquad [2]$$

energy conservation law:

$$\mathrm{d}\left\{\left(e + \frac{P}{\rho} + \frac{1}{2}u^2\right)\rho u A\right\} + \mathrm{d}Q = 0, \qquad [3]$$

where A is the cross sectional area of the pipe, P is the pressure, u is the velocity, e is the internal energy and Q is the energy introduced externally. To study the propagation of one-dimensional waves, consider the model shown in figure 1. A coordinate is fixed with the pressure wave; upstream is denoted by suffix 1 and downstream by suffix 3. Integrating [1], [2] and [3] from state (1) to (3), we have

$$\rho_1 u_1 r_1^2 = \rho_3 u_3 r_3^2, \qquad [4]$$

$$\rho_1 u_1^2 r_1^2 = \rho_3 u_3^2 r_3^2 + \int_1^3 r^2\,\mathrm{d}P, \qquad [5]$$

$$\left(c_v T_1 + \frac{P_1}{\rho_1} + \frac{u_1^2}{2}\right) + \frac{Q_1}{\rho_1 u_1 r_1^2 \pi} = \left(c_v T_3 + \frac{P_3}{\rho_3} + \frac{u_3^2}{2}\right) + \frac{Q_3}{\rho_3 u_3 r_3^2 \pi}, \qquad [6]$$

where r is the radius of the pipe, T is the temperature and c_v is the specific heat at constant volume. The radius of the pipe changes from r_1 to r_3 corresponding to internal pressure. The relation between the internal pressure and the pipe radius is given by the Hooke's law,

$$\frac{r_3 - r_1}{r_1} = \epsilon_3 = \frac{C r_1}{E \cdot t}(P_3 - P_1). \qquad [7]$$

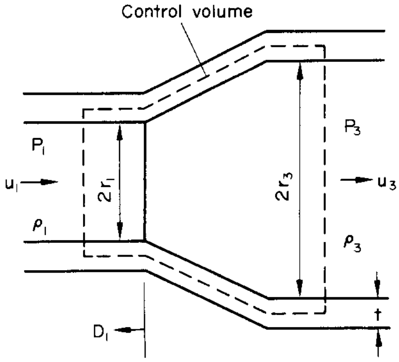

Figure 1. Flow model.

Here, E is the longitudinal elastic coefficient of the pipe material, ϵ is the strain, and t is the pipe wall thickness. The value of the coefficient C is dependent on the boundary conditions at either end and is given as follows (Kobori 1961):

one end fixed and the other free:

$$C = 1 - \frac{\mu}{2},$$

both ends fixed: [8]

$$C = 1 - \mu^2,$$

where μ is Poisson's ratio. The necessary condition for [7] is

$$|\epsilon_3| \ll 1, \text{ and thus } \Delta P \ll E\left(\frac{t}{Cr_1}\right). \quad [9]$$

In regard to Q in [6], considering an adiabatic wall, $Q_1 = 0$, and Q_3 is equal to the elastic energy,

$$Q_3 \doteqdot \epsilon_3 \frac{\Delta P}{\rho}, \ \Delta P = P_3 - P_1. \quad [10]$$

However, Q_3 is small compared with the pressure work,

$$\epsilon_3 \frac{\Delta P}{\rho} \ll \frac{P}{\rho}, \text{ and thus } \Delta P^2 \ll E \cdot P \cdot \left(\frac{t}{Cr_1}\right). \quad [11]$$

It is necessary for the pressure difference ΔP before and after the pressure waves to satisfy [11] or [9]. Generally, as $P \ll E(t/Cr_1)$, ΔP is only required to satisfy [11]. For pipe materials E is of order 100 GPa and $t/Cr_1 = O(0.1)$, so within $\Delta P = 10$ MPa, [11] and [9] are satisfied fairly well; and Q_1 and Q_3 in [6] can be neglected.

The integration of the second term of the right side of [5] has a value dependent on relaxation phenomena such as pressure waves. In this study we give attention only to the equilibrium conditions before and after the pressure wave. Consequently, the exact value of the second term of [5] is not known, but in general the following relation can be accepted:

$$r_1^2(P_3 - P_1) < \int_1^3 r^2 \cdot dP = r_2^2(P_3 - P_1) < r_3^2(P_3 - P_1), \quad [12]$$

where r_2 is the tube radius at the center of the pressure wave. If the width of the pressure wave is very thin, r_2 is considered to be equal to r_1. Putting $\epsilon_2 = (r_2 - r_1)/r_1$, from [7], we have

$$0 < |\epsilon_2| < |\epsilon_3| \ll 1. \quad [13]$$

Using [13], [4] and [5] are arranged as:

$$D_1^2 = u_1^2 = \left(\frac{1 + 2\epsilon_2}{1 + 2\epsilon_3(\rho_1/\Delta\rho)}\right) \cdot \frac{\rho_3}{\rho_1} \cdot \frac{\Delta P}{\Delta\rho} \quad [14]$$

where $\Delta\rho = \rho_3 - \rho_1$, and D_1 is the propagation speed of the pressure wave relative to the pipe. Considering [5] and [13],

$$D_1^2 = \frac{\rho_3}{\rho_1}\left\{\frac{\Delta\rho}{\Delta P} + \frac{2Cr_1}{E \cdot t}\rho_1\right\}^{-1}. \quad [15]$$

To obtain the value of $\Delta P/\Delta\rho$ in [15], the equations of mass conservation [4], momentum [5], energy [6] and state of the medium should be simultaneously solved. As the equations of state vary considerably for liquid, vapor or two-phase medium of vapor and liquid, in this study an isothermal change is assumed for the pressure wave unless specifically noted otherwise. This assumption is a good approximation, as the compressive work is small compared with the internal energy for the liquid. In the low void region, the internal energy of the liquid is large compared with that of the vapor and the isothermal change is again a good approximation.

2.2 *Pressure wave in the liquid phase*

$\Delta P/\Delta\rho$ in [15] is obtained by Taylor expansion:

$$\frac{\Delta\rho}{\Delta P}=\left(\frac{\partial\rho}{\partial P}\right)_T\left(1+\frac{1}{2}\frac{(\partial^2\rho/\partial P^2)_T}{(\partial\rho/\partial P)_T}\Delta P+\cdots\right). \qquad [16]$$

To obtain the sonic velocity of the isothermal change in the liquid phase, the derivatives on the right side of [16] should be obtained from the equation of state of liquid. The equation of state of water, if the specific volume is expressed as

$$v=f_0(T)+f_1(T)P+f_2(T)P^2$$

is given as above. For instance, $(\partial^2\rho/\partial P^2)/2(\partial\rho/\partial P)$ is calculated to be about 0·01 MPa^{-1} and if P is below 10 MPa, the second and higher order terms on the right side of [16] may be omitted, and:

$$\frac{\Delta\rho}{\Delta P}=\left(\frac{\partial\rho}{\partial P}\right)_T=\frac{1}{a_l^2}=\text{constant},\qquad D_l^2=\left(a_l^{-2}+\frac{2Cr_1\rho_l}{E\cdot t}\right)^{-1}, \qquad [17]$$

the subscript l denoting the liquid phase.

Therefore, the pressure wave in the water maintains the propagation velocity expressed by [17] independent of amplitude. Moreover, regardless of either the compression or expansion wave, the pressure wave retains the initial form of the wave front, a property far different from the shock wave in a gas.

2.3 *The pressure wave in the vapor–liquid two-phase flow*

2.3.1 *The case of mechanical equilibrium.* Consider that the vapor and the liquid move with the same speed, and that the two-phase medium behaves as a uniform single medium, and study the propagation of a pressure wave.

The density of the vapor–liquid two-phase medium is given as follows:

$$\rho=\alpha\rho_g+(1-\alpha)\rho_l, \qquad [18]$$

where α is a volume ratio of vapor (void fraction) and the subscript g denoting the vapor phase. The mass conservation equation for the vapor is

$$\alpha_1\rho_{g1}u_1r_1^2=\alpha_3\rho_{g3}u_3r_3^2. \qquad [19]$$

By the use of [18], [19] and [4], $\Delta\rho$ is given as

$$\frac{\Delta\rho}{\rho_1}=\frac{(1-\alpha_1)\cdot\dfrac{\Delta\rho_l}{\rho_{l1}}+\alpha_1\cdot\dfrac{\Delta\rho_g}{\rho_{g1}}+\dfrac{\Delta\rho_l}{\rho_{l1}}\cdot\dfrac{\Delta\rho_g}{\rho_{g1}}}{1+\left\{(1-\alpha_1)\dfrac{\Delta\rho_g}{\rho_{g1}}+\alpha_1\dfrac{\Delta\rho_l}{\rho_{l1}}\right\}}. \qquad [20]$$

Assuming the isothermal change

$$\left(\frac{\Delta\rho_g}{\Delta P}\right)_T = \frac{1}{RT}$$

$$\left(\frac{\Delta\rho_l}{\Delta P}\right)_T = \frac{1}{a_l^2}. \qquad [21]$$

Substituting [21] and [20] into [15], the propagation velocity is

$$D_1^2 = \frac{\rho_3}{\rho_1}\left[\left\{\frac{\frac{1-\alpha_1}{\rho_l a_l^2} + \frac{\alpha_1}{P} + \frac{\Delta P}{P}\cdot\frac{1}{\rho_l a_l^2}}{1+\left(\frac{1-\alpha_1}{P} + \frac{\alpha_1}{\rho_l a_l^2}\right)\Delta P}\right\}\cdot\rho_1 + \frac{2Cr_1}{E\cdot t}\cdot\rho_1\right]^{-1}. \qquad [22]$$

The second term of the right hand side of this equation expresses the effect of elastic deformation of the pipe. In order to examine this effect, we consider the case of small amplitude of ΔP, $\Delta P \to 0$, and [22] is,

$$D_1^2 = \left[\left\{\frac{\alpha_1}{P} + \frac{1-\alpha_1}{\rho_l a_l^2} + \frac{2Cr_1}{E.t}\right\}\{\alpha_1\rho_g + (1-\alpha_1)\rho_l\}\right]^{-1} \qquad [23]$$

which is depicted in figure 2. The vertical axis is the propagation velocity of the pressure wave D_1, and the horizontal axis is the void fraction α, or $(1-\alpha)$. In this figure for an air–water system, when $P = 0.1$ MPa and $2Cr_1/t = 10$, the changes of the sonic velocity of liquid and the longitudinal elastic coefficient E of the elastic pipe are shown. For void fraction below 10^{-2} the propagation speed decreases in accordance with a decrease of the elastic coefficient. However, for the void fraction above 10^{-2}, the propagation velocity does not depend on a_l and E. Because an isothermal change is considered rather than an adiabatic change for $\alpha \to 1$, the propagation velocity does not tend to the sonic velocity in air. As seen from this figure, in a small region such as $\alpha < 10^{-4}$, α can be obtained from the measurement of the sonic velocity. For the propagation velocity of a pressure wave in a liquid, it is necessary to completely degasify the liquid, or to make the void substantially small by increasing the pressure.

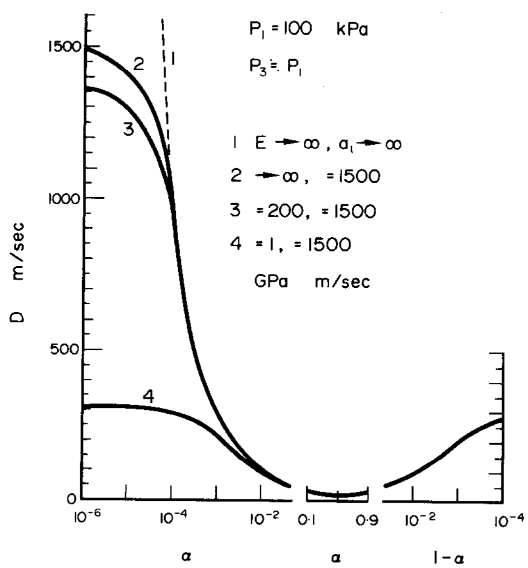

Figure 2. Sound velocity in two-phase flow. (Theory).

Limiting the discussion in the region of $\alpha_1 \gg (\Delta P/\rho_l)/a_l^2$, [22] becomes

$$D_1^2 = \frac{P_3}{\alpha_1(1-\alpha_1)\rho_l}, \tag{24}$$

and the effect of the elasticity of the pipe disappears. This equation shows that the propagation velocity D_1 is determined by the pressure P_3 after the pressure wave, and this pressure wave forms a shock wave. The applicable region of α_1 for the air–water system of $P_1 = 0.1$ MPa is about $\alpha_1 > 10^{-2}$.

2.3.2 *The case of a relative motion between two phases.* The preceding discussion was based on the assumption that the two phases move with the same speed. However, experimentally, the vapor and liquid flows which are moving with the same speed before the pressure wave do not always move with the same speed after the pressure wave. In order to study this effect, the equation of motion of vapor or liquid is required in addition to the equation of motion [2].

Neglecting the elasticity of the pipe, the equation of motion for vapor is (Landau 1960):

$$\mathrm{d}(\alpha\rho_g u_g^2) + W\,\mathrm{d}(u_g - u_l) + \alpha\,\mathrm{d}P = 0. \tag{25}$$

Before the pressure wave the vapor and liquid are assumed to move with the same speed $u_1 = u_{g1} = u_{l1}$, and [25] is integrated from state (1) to (3):

$$\alpha_1\rho_{g1}u_1(u_1 - u_{g3}) - W(u_{g3} - u_{l3}) = \alpha_2(P_3 - P_1). \tag{26}$$

The second term of the left hand side is an acceleration term of a virtual mass. The virtual mass W depends strongly on the flow conditions of the two-phase flow. For a bubbly flow where bubble intervals are sufficiently large that interference between bubbles can be neglected, $W = \alpha_1\rho_{l1}u_{g1}/2$; for a mist flow where the droplets are sufficiently separated, $W = (1-\alpha_1)\rho_{g1}u_{l1}/2$; when the vapor and the liquid are completely mixed, $W \to \infty$; when the motions of vapor and fluid are completely independent, $W = 0$. When $W = 0$, the void fraction before and after the pressure wave shows no change ($\alpha_1 \fallingdotseq \alpha_3$) and in this case $D_1 = P_3/\rho_{g1}$, and the pressure wave propagates with a speed almost equal to the sonic velocity in vapor. When $W \to \infty$, $u_{g3} = u_{l3}$, and the propagation velocity agrees with the solution for the homogeneous model mentioned above, given by [24]. Thus the propagation velocity varies strongly with the virtual mass. The value of the virtual mass is so dependent on the flow pattern that it can neither be expressed as a function of void fraction alone, nor is value known for all cases. Therefore we proceed with our discussion assuming $W \to \infty$.

2.4 *Reflection of pressure wave*

Consider a case in which the pressure wave of such a propagation velocity as given by [24] reflects at the rigid wall surface, and the condition after the reflection wave is denoted by suffix 5. By [22], the propagation velocity of the incident wave and the reflected wave are respectively

$$D_1^2 = \frac{P_3}{(1-\alpha_1)\alpha_1}, \qquad D_3^2 \fallingdotseq \frac{P_5}{(1-\alpha_3)\alpha_3}, \tag{27}$$

and the void fraction after the incident wave is,

$$\alpha_3 = \frac{\alpha_1\cdot(P_1/P_3)}{1-\alpha_1+(P_1/P_3)\alpha_1}. \tag{28}$$

As the difference $(u_1 - u_3)$ between the pressure wave velocity before reflection and the particle velocity at the rigid wall, and the particle velocity at the rigid wall after the reflected wave $(u_3 - u_5)$ must be equal, the pressure after the reflection is obtained as:

$$\frac{P_3}{P_1} = \frac{P_5}{P_3}, \qquad \frac{P_3}{P_1} = \theta_1, \qquad \frac{P_5}{P_3} = \theta_3, \qquad \theta_1 = \theta_3, \tag{29}$$

and the pressure ratios become equal. On the other hand, the pressure after the reflection of the pressure wave propagating in the fluid with the propagation velocity as shown in [17] is,

$$P_3 - P_1 = P_5 - P_3, \qquad \theta_3 + \frac{1}{\theta_1} = 2, \tag{30}$$

and the pressure differences are equal.

2.5 *The pressure wave of isentropic change in the two-phase flow*

The preceding discussion is based on the assumption of isothermal change. If the relaxation time of the energy transport phenomenon between the phases is sufficiently long compared to the time corresponding to the pressure change of the whole system, the temperature of the two phases is not always equal. If we consider that the vapor changes isentropically, we have,

$$\mathrm{d}s = \mathrm{d}e_g + P\,\mathrm{d}\frac{1}{\rho_g} = 0, \qquad P/\rho_g^{\kappa} = \text{constant}, \tag{31}$$

where κ is the specific heat ratio. If this relation is used instead of [21], from [15] we get,

$$D_{1s}^2 = \frac{P_3}{\rho_l(1-\alpha_1)\alpha_1}\left\{\frac{1 - P_1/P_3}{1 - (P_3/P_5)^{1/\kappa}}\right\}, \tag{32}$$

and the velocity of the reflected wave D_{3s}, the void fraction α_{3s} after the incident wave, and the relation of the pressures before and after reflection at the rigid wall are given as follows,

$$D_{3s}^2 \doteqdot \frac{P_5}{\rho_l(1-\alpha_3)\alpha_3}\left\{\frac{1 - P_3/P_5}{1 - (P_3/P_5)^{1/\kappa}}\right\}, \tag{33}$$

$$\alpha_{3s} = \frac{\alpha_1(P_1/P_3)^{1/\kappa}}{1 - \alpha_1 + (P_1/P_3)^{1/\kappa}\alpha_1}, \tag{34}$$

$$\theta_3^{1/\kappa}\left(\frac{\theta_1^{1/\kappa} - 1}{\theta_3^{1/\kappa} - 1}\right) = \theta_1\left(\frac{\theta_3 - 1}{\theta_1 - 1}\right), \tag{35}$$

and in the isentropic change, the wave velocity is faster than that in the isothermal change by

$$\left[\left\{1 - \frac{P_1}{P_3}\right\} \Big/ \left\{1 - \left(\frac{P_1}{P_3}\right)^{1/\kappa}\right\}\right]^{1/2}.$$

This value corresponds to the ratio of the propagation velocities in the isentropic change and the isothermal change in the vapor. On the pressure wave in the vapor, an isenthalpic rather than an isentropic change occurs.

3. EXPERIMENTS

The experimental apparatus consists of a semiconductor pressure gauge, a bubble generation device and a recorder as shown in figure 3. The vertical shock tube full of water constitutes

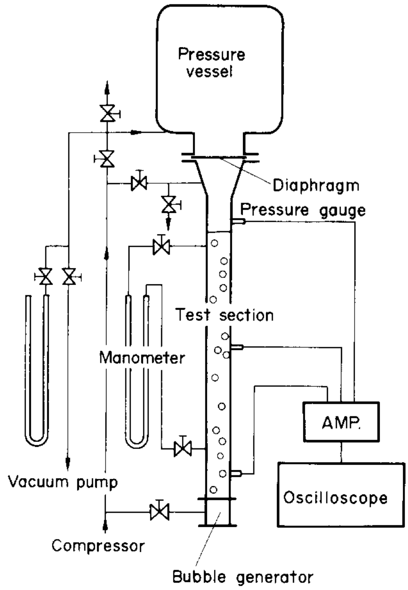

Figure 3. Experimental apparatus.

the test section. The pressure in the upper and lower portions of the test section can be varied by valve operation. Either a pressure or expansion wave can be propagated into the test section by puncturing a diaphragm. The upper part is made sufficiently large in order to permit use as a damping tank. For the test section, a steel pipe of 40 mm outer diameter, 24 mm inner diameter and 1850 mm long is used, minimizing the effect of elastic deformation. The void fraction in the pipe is measured by the static pressure difference at the upper and lower pressure gauges and the water head difference before and after the vapor bubble flow.

The two-phase bubbly flow was obtained by blowing high pressure air or high pressure nitrogen through the porous plate placed at the bottom part of the measuring section. The vapor bubbles are of diameter of 0.5–3.0 mm and vary with void fraction. However, by use of a surfactant the radius of vapor bubbles could be changed to some extent. The maximum void fraction obtainable by this experimental apparatus is 20 per cent and above this void fraction the flow pattern changes from a vapor bubble flow to a slug flow.

The propagation velocity of the pressure wave and the pressure amplitude were obtained from the traces on a storage oscilloscope, and pressure waves were measured by semiconductor pressure gauges. The trigger signal for oscilloscope was the breaking of the diaphragm detected by a piezoelectric element. Diaphragms consisted of 0.1–0.3 mm aluminum films. The film thickness determined the amplitude of the pressure wave.

For a preparatory experiment, the velocity and reflection ratio of the pressure wave propagating in water were measured, and the range of pressure difference was about 0.01–0.4 MPa. As explained in the theoretical analysis, since the pressure wave in the water progresses retaining the initial pressure wave form, a pressure wave of the step form is not obtained, and the wave keeps the form with the turbulence generated at the time of break of the diaphragm. In order to minimize this effect, it is necessary to make the distance between the diaphragm and the top surface of the liquid column sufficiently wide and adjust the pressure wave form in the vapor. The experimental result shows that if the diameter of the damping tank is greater than about 30 times the pipe diameter the pressure wave becomes almost of the step form. Due to its high velocity compared to that of a gas, the pressure wave in the liquid is liable to be affected by vibration of the pipe. The experimental result for the propagation velocity of the pressure wave in the water after degasing air is shown in figure 4. An almost constant propagation velocity 1500 m/sec was obtained regardless of the pressure amplitude through the pressure amplitude range of

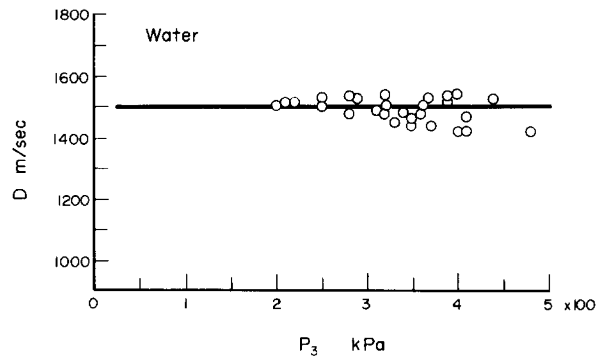

Figure 4. Shock speed in water.

0.1–0.4 MPa. This result agrees with the experimental result reported in the past on the sonic velocity in water.

Below the pressure difference of 0.1 MPa, a thin pressure wave was not obtained. The experimental result for the reflection ratio is shown in figure 5. The vertical axis is the pressure ratio before and after the reflected wave, the horizontal axis is the pressure ratio before and after the incident wave, and the solid line in the figure is the theoretical solution given by [30] for the rigid wall reflection. The experimental values and the theoretical values expressed by circle marks show good agreement, but the experimental values of P_5/P_3 are somewhat smaller than the model prediction. The value of the reflection ratio in infinitesimal amplitude theory is about 0.94 for the water–steel system. We expect that the reason that the experimental value is slightly smaller than the theoretical value is because the steel pipe is not a completely rigid body against water.

One example of the results of measurement on the pressure change in the two-phase flow by a storage oscilloscope is shown in the lower half of figure 6. The upper half of the figure is an x–t line of the pressure wave in the two-phase flow and is correlated to the signals of the oscilloscope. In figure 6 the upper curve of the signals shows the pressure change measured by a semiconductor pressure gauge set on the top part of the pipe, and the lower curve is that by a semiconductor pressure gauge set at the lower part. The pressure wave crest has completely a step response showing that the pressure wave in the two-phase flow is a shock wave. The behavior of propagation of the pressure wave is explained by the upper half of figure 6. The result of this measurement proved that in the two-phase flow the velocity of the reflected wave was much faster compared with that of incident wave and the pressure after reflection was much larger compared with that before the reflected wave. Based on results such as shown in figure 6, the velocities of incident and reflected waves are obtained and are shown against the void fraction α_1, before the arrival of incident wave in figure 7. The solid line in the figure indicates the

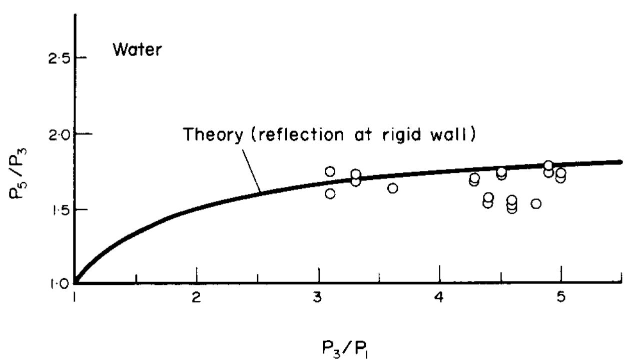

Figure 5. Reflection of pressure waves in water.

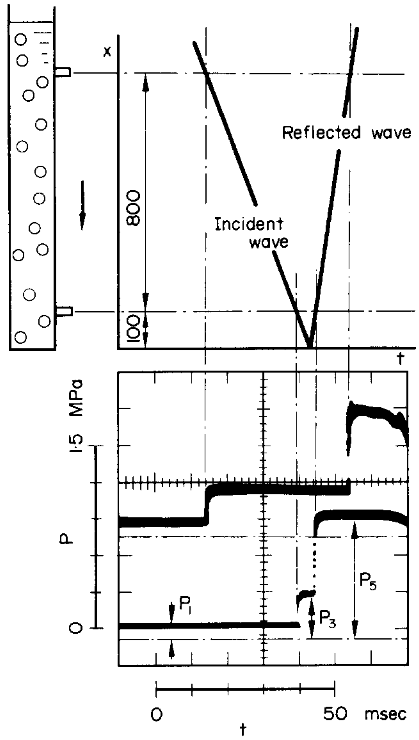

Figure 6. One example of measurement on pressure change in two-phase flow.

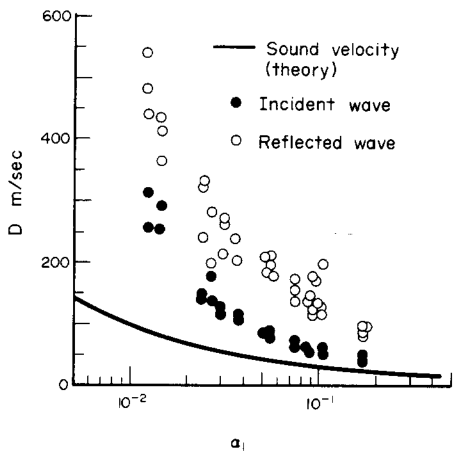

Figure 7. Shock speed in two-phase flow.

propagation velocity of small amplitude pressure waves (sound waves) of isothermal change in the atmosphere pressure vs the void fraction before the incident wave expressed by

$$D^2 = \frac{P_1}{\alpha_1(1-\alpha_1)\rho_l}.$$

In the figure the circle marks are propagation velocities of reflected waves and black marks are of incident waves. The propagation velocity of incident waves is faster than that of sonic waves, and the propagation velocity of reflected waves is much faster than that of incident waves. We consider this difference is due to the finite amplitude of the pressure wave, and for the incident waves, the propagation velocity, D_{exp}, obtained from the experiment is modified as $D_{exp}^2 \cdot P_1/P_3 = D^2$ according to [27]. D is shown with white circles in figure 8. This value is for the

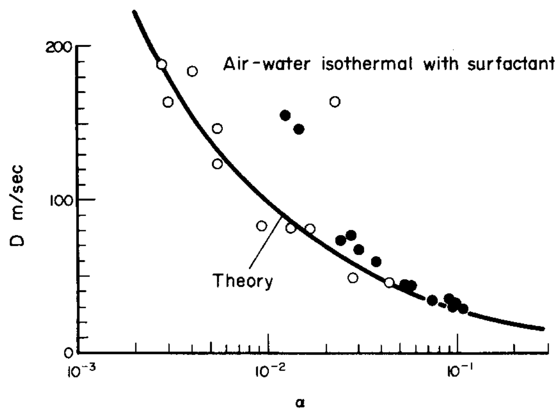

Figure 8. Modified sound velocity in two-phase flow. (Isothermal).

incident wave, and if for the reflected wave the similar modification is made by use of the pressure and the void fraction, α_3, almost the same result is obtained. To compare the experimental result with the isentropic value, another modification is made:

$$D^2_{\text{exp}} \cdot \frac{P_1}{P_3} \cdot \frac{1-(P_1/P_3)^{1/\kappa}}{1-P_1/P_3} = D^2.$$

This D is shown in figure 9. It is not possible to distinguish at present from figures 8 and 9, to which analytic model the experimental value is well correlated. For small void fractions the propagation velocity experimental value, black circles in figure 8, is larger than the theoretical value. The accuracy of the measurement is considered to be considerably deteriorated for a low void. In the case of the small void fraction, a decrease of numbers of bubbles of constant bubble radius causes invalidation of the assumption of a homogeneous mixture of gas and liquid, and a large difference is seen in the lower void region between theoretical and experimental values. Consequently, such experiments are performed decreasing bubble diameters by adding a surfactant and improving the homogeneity of flow by the increase of bubble number. The experimental results by this procedure are shown with white circles in figure 8. They are to some extent dispersed but show a good agreement with the isothermal propagation velocity $D^2 = P_3/\{\rho_l(1-\alpha_1)\alpha_1\}$. It is considered that the dispersion of experimental values of white circles is due to void fraction measurement error.

From the results mentioned above, it is considered that the propagation velocity of finite amplitude compression pressure waves depend on the condition immediately after the pressure wave; the bubbles are compressed in the downflow of the wave surface, the void fraction becomes small, and the sonic velocity increases as seen from figure 2, which determines the

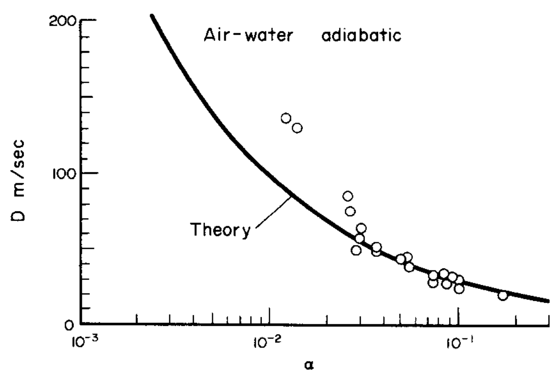

Figure 9. Modified sound velocity in two-phase flow. (Adiabatic).

propagation velocity of pressure waves. From this consideration it is assumed that there is a difference in propagation phenomenon between the compression wave and the expansion wave in two-phase flow.

The pressure change by reflection at the bottom end of the shock tube is shown in figure 10. The vertical axis is the pressure ratio before and after the reflected wave, the horizontal axis is the pressure ratio before and after the incident wave, and the solid line indicates the theoretical solution of [29] about the reflection ratio at the rigid surface when an isothermal change is assumed. The experimental value is smaller than the theoretical value, but different from the case of liquid; many results of the pressure ratio of P_5/P_3 above 2.0 are included. These results, normalized by the theoretical value and P_5P_1/P_3^2 obtained from the experimental value in figure 8 against void fraction, are shown in figure 11, which proves an increase of reflection ratio in accordance with void fraction. Whether the reflection surface is considered to be a rigid surface or not depends on the ratio of impedances of the medium propagating the pressure wave and of the material of the reflecting surface. Sound impedance z is given by multiplying the sonic velocity a in the medium by density ρ of the medium. The difference from the theory in figure 11 for the small α region is due to imperfect rigid reflection because of a low sonic impedance, as the reflection plate at the bottom edge is a porous plate. That is, the lower the void fraction, the higher the sound impedance of the two-phase medium, and the porous plate cannot be treated as a rigid body; however, due to the increase of void fraction it is assumed the sonic impedance decreases below that of the porous plate and the condition of reflection approaches to the rigid reflection.

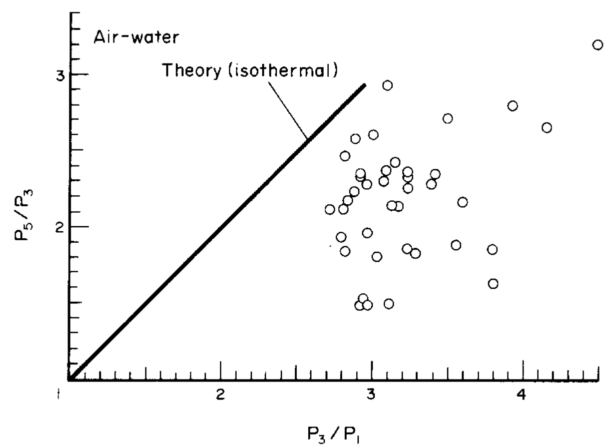

Figure 10. Pressure change by reflection.

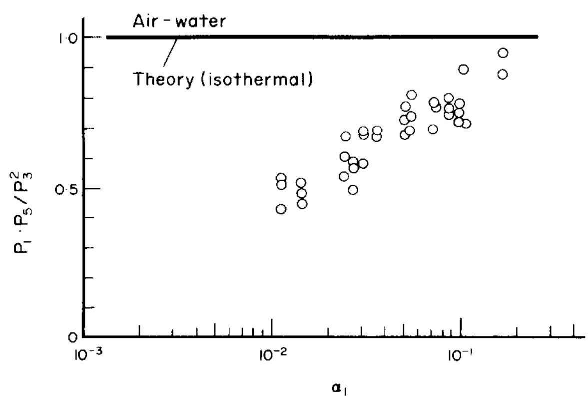

Figure 11. Pressure change by reflection.

4. CONCLUSION

The finite amplitude pressure wave propagating in a liquid or a gas and liquid two-phase medium in an elastic pipe was studied theoretically and experimentally. The effect of the elastic deformation of the pipe on the propagation velocity of pressure waves and the propagation performance of the pressure wave in the liquid and in the gas–liquid two-phase flow, was clarified.

(1) The effect of pipe elasticity appears below 1 per cent of void fraction in the gas–liquid two-phase flow at atmospheric pressure.

(2) The propagation velocity of finite amplitude forms shock waves with regard to the pressure wave, is given by $D_1 = [P_3/\{(1-\alpha_1)\alpha_1\rho_l\}]^{1/2}$.

(3) For reflection at a rigid surface, the pressure in the liquid after reflection is $P_5 = 2P_3 - P_1$, while in the gas–liquid two-phase flow we have $P_5 = P_3^2/P_1$.

(4) The experimental value for the propagation velocity of the pressure wave in the liquid shows a good agreement with those reported in the past, and the experimental value of the reflection ratio at the bottom end of the shock tube is a little smaller than the rigid reflection; however it is explained by considering that the material on the bottom end is not a perfectly rigid body.

(5) The propagation velocity of compression pressure waves in the gas–liquid two-phase medium agrees closely with the theoretical value, but with decrease of void fraction and large bubble diameters, the experimental value exceeds the theoretical value. This must be due to the deterioration of homogeneity of bubbly two-phase flow. Adding a surfactant to the liquid and decreasing bubble diameters, down to about 10^{-3} void fraction we find a good agreement between the theoretical value and the experimental value within void fraction measurement error.

(6) The reflection ratio of the pressure wave at the bottom edge of the shock tube for the gas–liquid two-phase medium varies strongly with void fraction. This is because the sonic impedance of the gas–liquid two-phase medium varies with void fraction.

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Résumé—On s'intéresse à une onde de pression d'amplitude finie se propageant dans un milieu diphasique gaz–liquide ou dans le fluide contenu dans un tube élastique. On étudie l'influence de l'élasticité de la conduite sur la vitesse de propagation de l'onde de pression. De même, on traite le problème des ondes de pression d'amplitude finie se déplaçant dans un écoulement diphasique, en prenant en compte la variation de taux de vide due à l'augmentation de pression. La vitesse de propagation d'une onde de choc diphasique est également étudiée. Les comportements lors de la reflexion de l'onde de pression sur une paroi rigide sont analysés et comparés à ceux du gaz ou du liquide purs. Les résultats sont comparés aux données expérimentales pour des ondes de pression se propageant en écoulement diphasique dans un tube à choc vertical.

Auszug—Druckwellen endlicher Amplitude, die sich in einem Gas–Fluessigkeits-Medium oder in einer Fluessigkeit in einem elastischen Rohr fortpflanzen, werden behandelt. Der Einfluss der Rohrelastizitaet auf die Fortpflanzungsgeschwindigkeit der Druckwelle wird untersucht. Ebenso werden in einer Zweiphasenstroemung verlaufende Druckwellen endlicher Amplitude betrachtet, unter Beruecksichtigung der durch den Druckanstieg hervorgerufenen Aenderung des Hohlraumanteils. Die Fortpflanzungsgeschwindigkeit der

Zweiphasen-Stosswellen wird auch untersucht. Das Reflexionsverhalten der Druckwelle an einer festen Wand wird analysiert und mit dem reiner Gase oder Fluessigkeiten verglichen. Die Ergebnisse werden mit Versuchsdaten von Druckwellen-Fortpflanzung in Zweiphasenstroemung im vertikalen Stossrohr verglichen.

Резюме—Нами рассмотрена волна давления с конечной амплитудой, распространяющаяся в газе и жидкой среда или в жидкости, заключенной в эластичной трубе. Нами изучалось влияние эластичности трубы на скорость распространения волны давления. Наряду с этим рассмотрены волны давления с конечной амплитудой, распространяющиеся в двуфазном потоке с учетом изменения доли пустот, вызванного увеличением давления. Исследована также скорость распространения двуфазно ударной волны, проанализировано процесс отражения волны давления от жесткой стенки и сравнено подобным в чистом газе и чистой жидкости. Разультаты сравнены с экспериментальными данными о волных давления, распространяющихся в двуфазном потоке вертикальной ударной трубы.